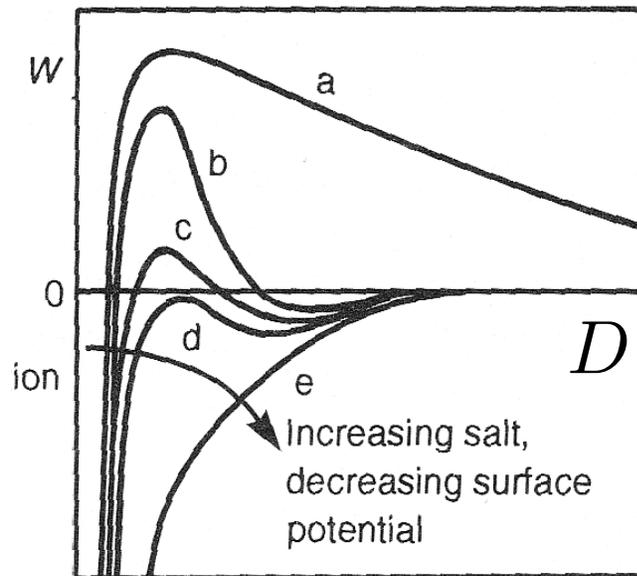


Colloidal flocculation

Flocculation

Reduced barrier



Israelachvili, *Intermolecular and Surface Forces*, 3rd ed., Academic Press, New York, 2007.

Double layer and vdw interactions (DLVO, 1941 & 1948)

$$U_e \approx 64k_B T a n_\infty \gamma^2 \kappa^{-2} e^{-\kappa D}$$

$$U_{\text{vdw}} \approx \frac{aA_H}{12D} \quad D \ll a$$

Critical counter-ion concentration for flocculation:

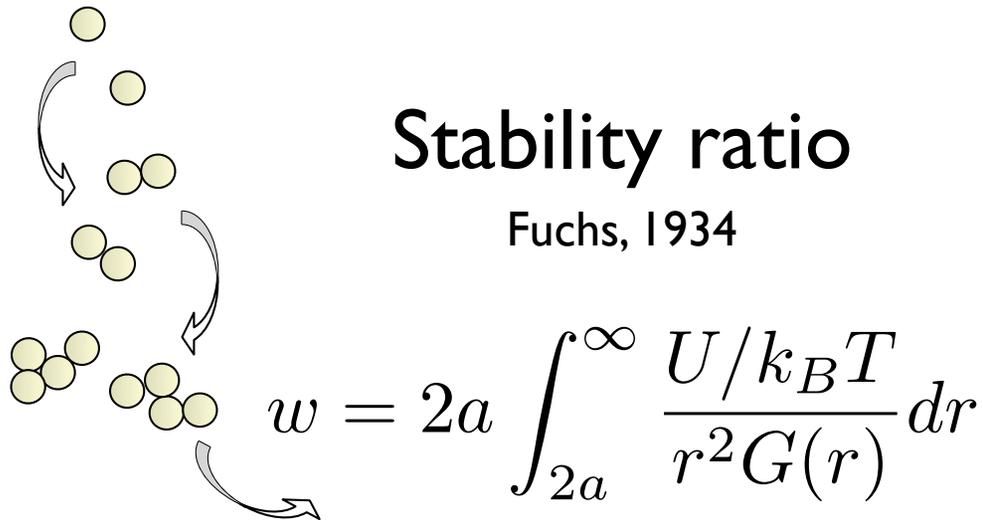
$$n_{\text{crit}} = \frac{49.6}{z^6 l_b^3} \left[\frac{k_B T}{A_H} \right] \tanh^4 \left(\frac{ez\psi_s}{4k_B T} \right)$$

$$l_b = e^2 / 4\pi\epsilon\epsilon_0 k_B T \quad \text{Bjerrum length}$$

Doublet formation

Stability ratio

Fuchs, 1934



$$w = 2a \int_{2a}^{\infty} \frac{U/k_B T}{r^2 G(r)} dr$$

$$w = J_0/J$$

Doublet formation

$$t_p = \frac{\pi \mu a^3 w}{\phi k_B T}$$

Doublet formation times

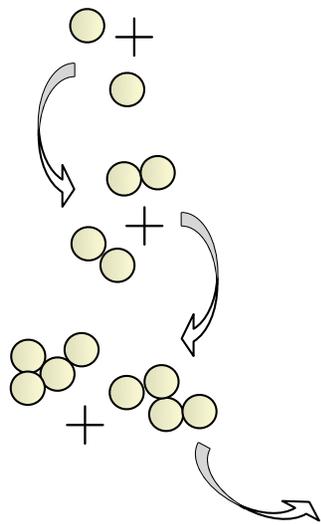
$$w = 1$$

t_p	$\phi = 10^{-5}$	$\phi = 10^{-1}$
$a = 0.1 \mu m$	70s	7ms
$a = 1.0 \mu m$	20h	7s

W. B. Russel, D. A. Saville, and W. R. Schowalter, Colloidal Dispersions (Cambridge University Press, New York, 1989).

Flocculation kinetics

Smoluchowski, 1917



Particle flux

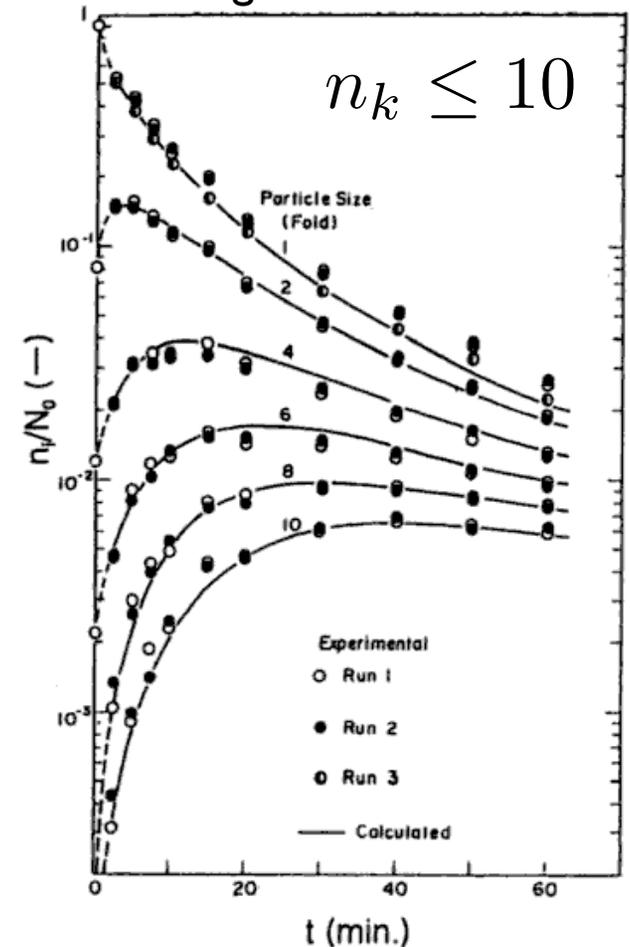
$$J_{ij} = \frac{2k_B T}{3\mu} (a_i + a_j) \left(\frac{1}{a_i} + \frac{1}{a_j} \right) \frac{n_i n_j}{w_{ij}}$$

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{i=1, j=k-i}^{k-1} J_{ij} - \sum_{i=1}^{\infty} J_{ki}$$

$$n_k(t) = n_0 \frac{(t/t_p)^{k-1}}{(1 + t/t_p)^{k+1}}$$

$$t_p = 3\mu w / 4n_0 k_B T$$

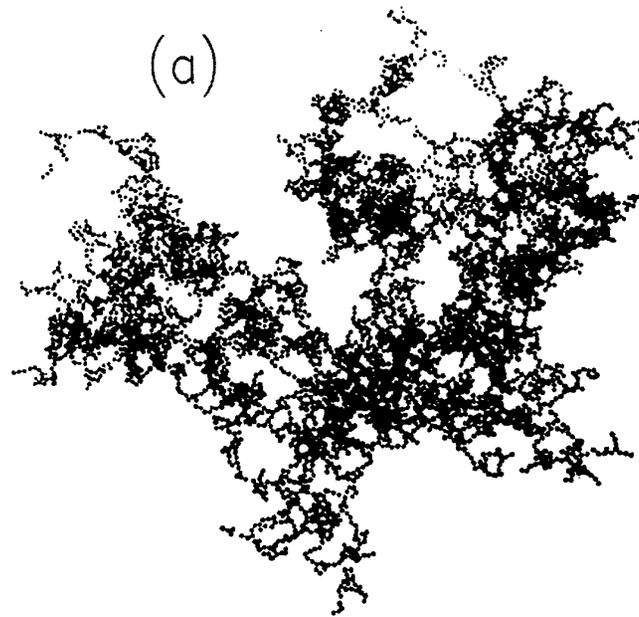
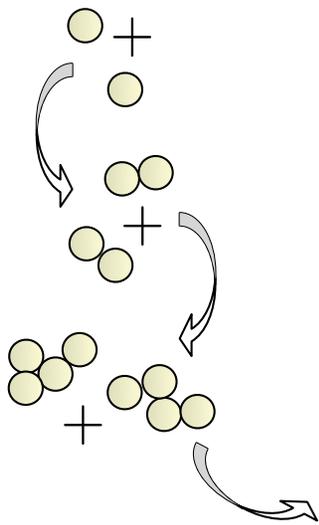
Polystyrene latex flocculation
using Coulter counter



Higashitani & Matsuno, 1979.

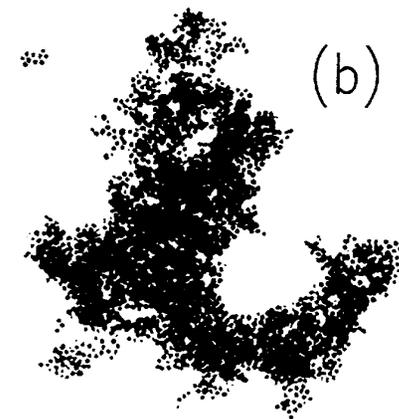
Floc structure

$$\xi \sim ak^{d_f} \quad d_f \text{ Fractal dimension}$$



Diffusion limited
 $d_f \approx 1.7 - 1.8$

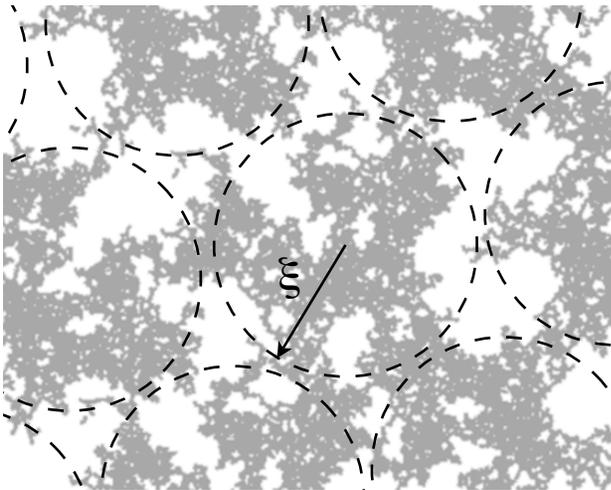
7.2nm Au particles



Reaction limited
 $d_f \approx 2.0 - 2.2$

Percolation

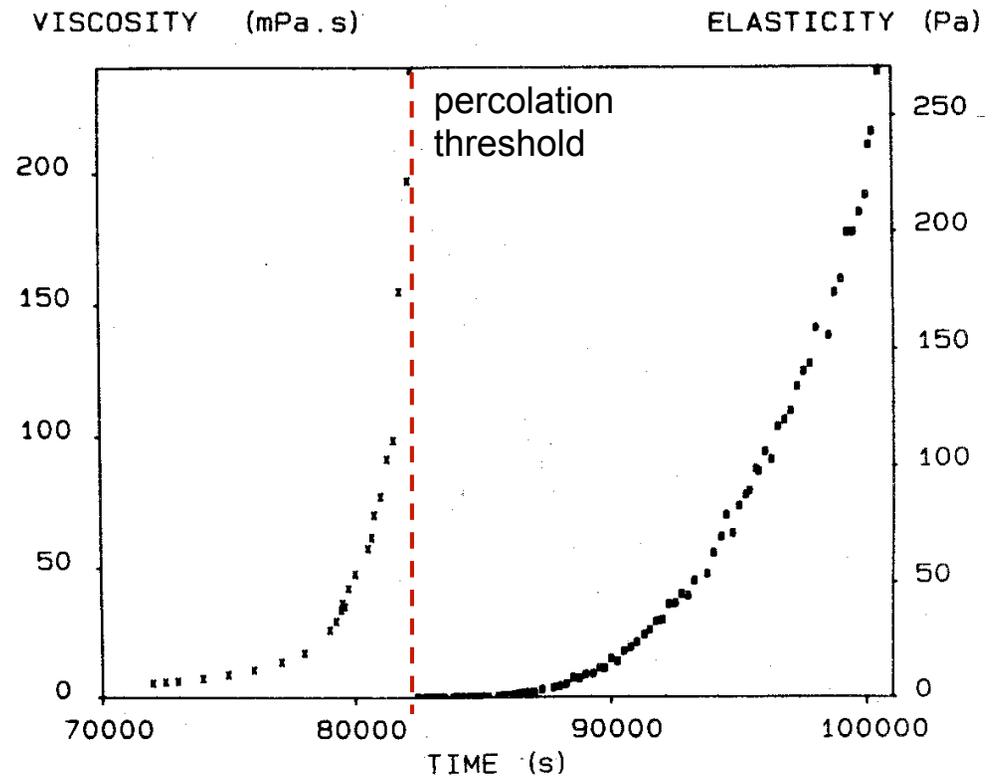
Flocs close-packed:



$$\phi_k \approx \phi_{\text{rcp}} = 0.64$$

$$\phi_k = \phi_{\text{rcp}} k^{1-3/d_f}$$

$$\xi = a(\phi_{\text{rcp}}/\phi)^{1/(3-d_f)}$$



Gauthier-Manuel et al., *J. Phys.* **48**:869, 1987.

Gel rheology

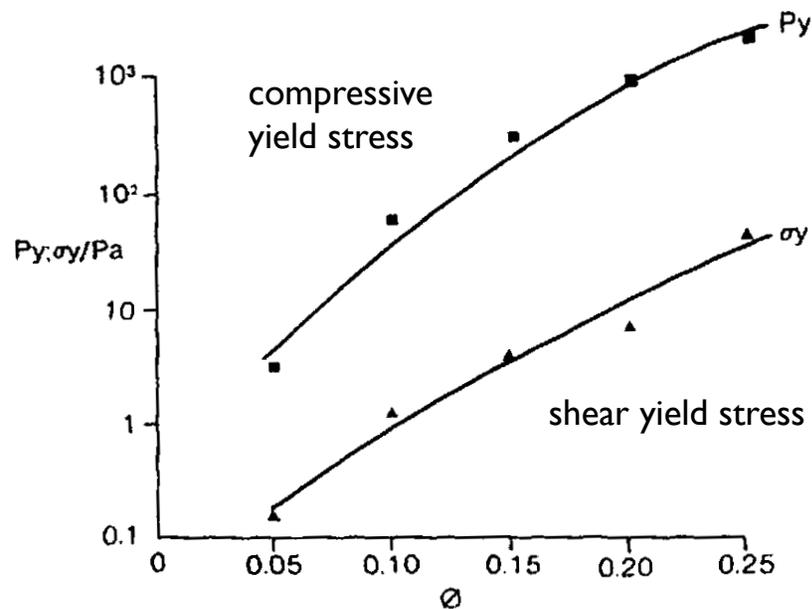
Gel rheology

- Elastic modulus, yield stress, yield strain
- Model and “real” systems:
 - polystyrene latices in water
flocculated (salts) or with non-adsorbing polymer
 - poly(methyl methacrylate) in organic solvents
with non-adsorbing polymer – “depletion” gels
 - “organophilic” silica in organic solvents
with non-adsorbing polymer or “thermoreversible”
 - mineral suspensions
typically aqueous, flocculated by addition of salts

Gel rheology — flocculated polystyrene latices

Buscall, R. et al. "The rheology of strongly-flocculated suspensions." J. Non-Newtonian Fluid Mech. 24, 183–202 (1987).

Polystyrene latex in salt solutions



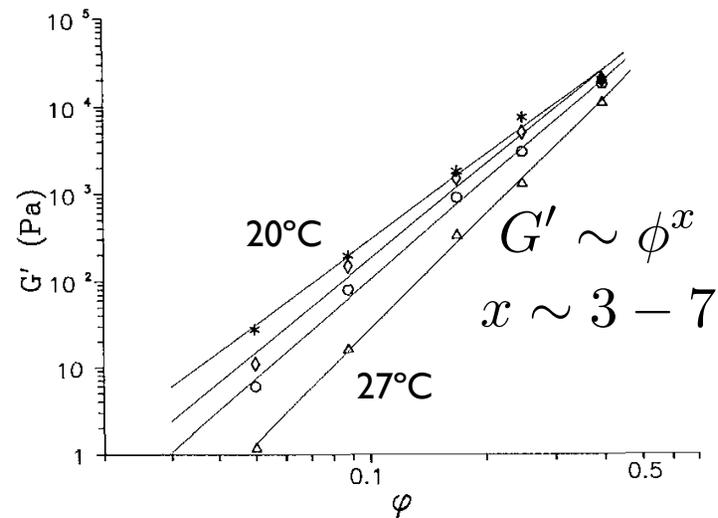
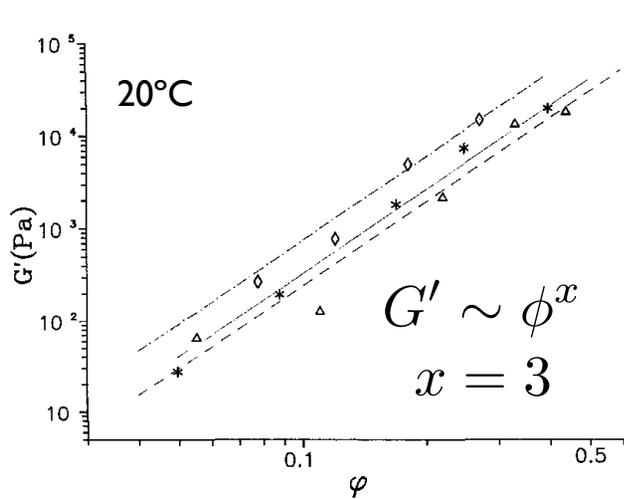
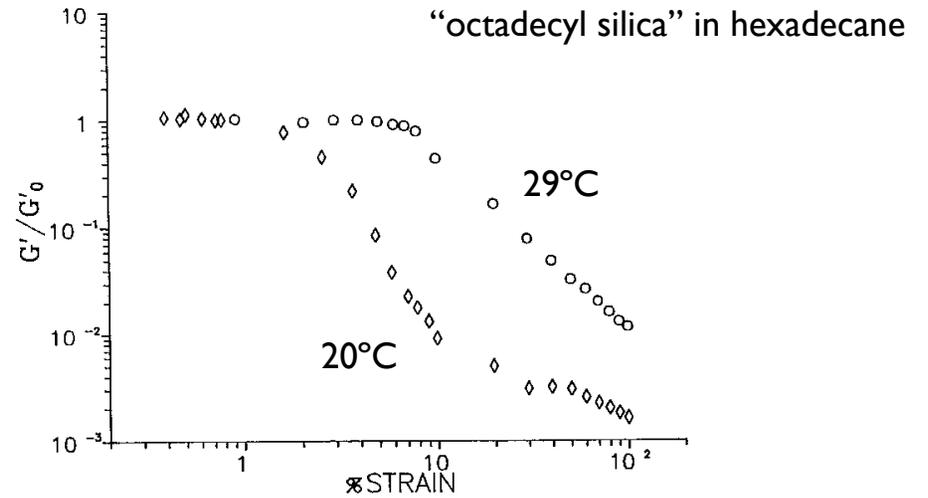
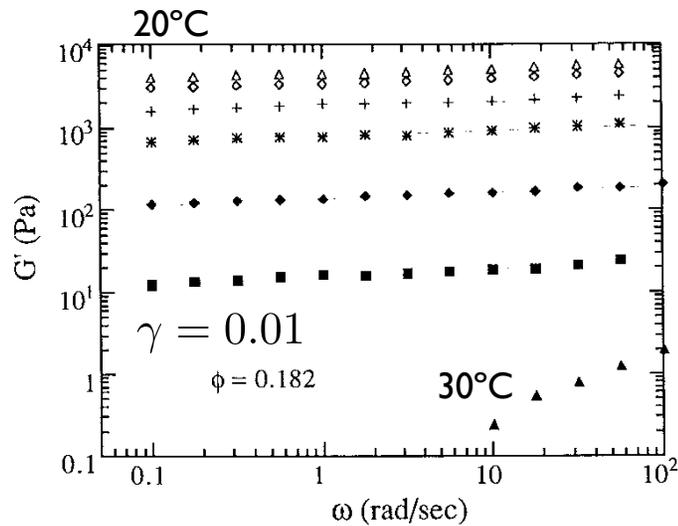
$$P_y \propto G \propto \phi^x$$

$$x \sim 4.0 - 5.0$$

Compression — dewatering and filtration

Gel rheology — Stöber organo-silica

Chen, M. & Russel, W. B. "Characteristics of flocculated silica dispersions." *J Colloid Interface Sci.* 141, 564–577 (1991).



Gel rheology — Stöber organo-silica

Rueb, C. J. & Zukoski, C. F., “Viscoelastic properties of colloidal gels.” J. Rheol. 41, 197–218 (1997).

“octadecyl silica” in decalin

$$G' \sim \phi^x$$

$$x = 4.4 - 5.6$$

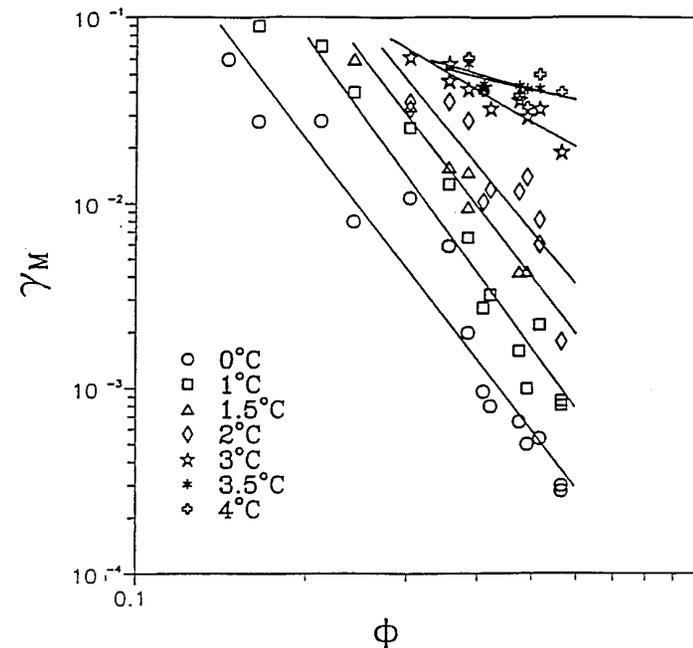
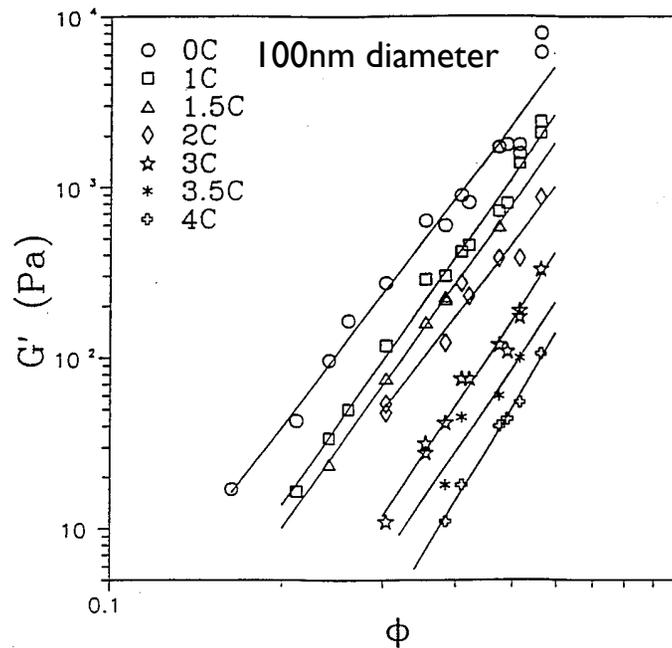


TABLE II. Power-law exponents for gel mechanical properties.

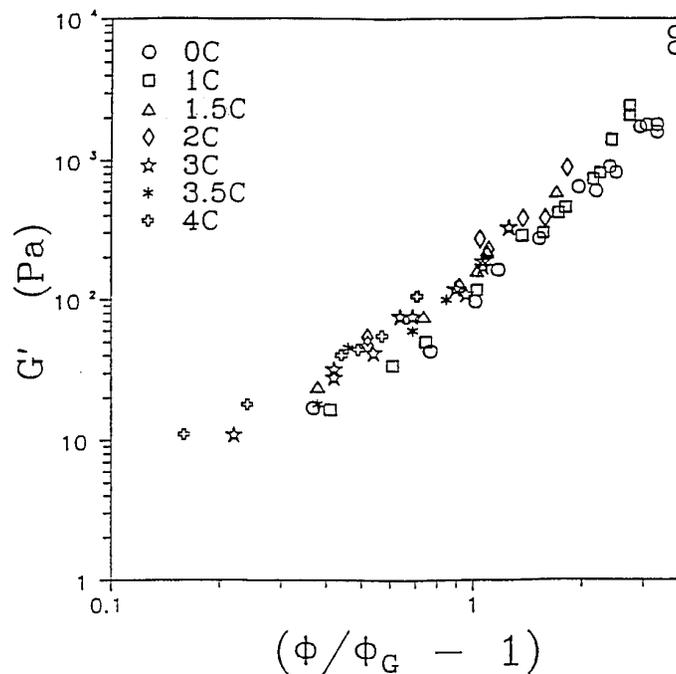
Temp	0 °C	1 °C	1.5 °C	2 °C	3 °C	3.5 °C	4 °C
x^a	4.4	4.8	4.7	4.4	5.1	5.0	5.6
t^b	-4.0	-4.2	-3.9	-3.7	-1.8	-0.8	-0.7

^aPower-law exponent relating G'_∞ to volume fraction: $G'_\infty \sim \phi^x$. Uncertainties in x are ± 0.4 .

^bPower-law exponent relating γ_M to volume fraction: $\gamma_M \sim \phi^t$. Uncertainties in t are ± 0.2 .

Gel rheology — Stöber organo-silica

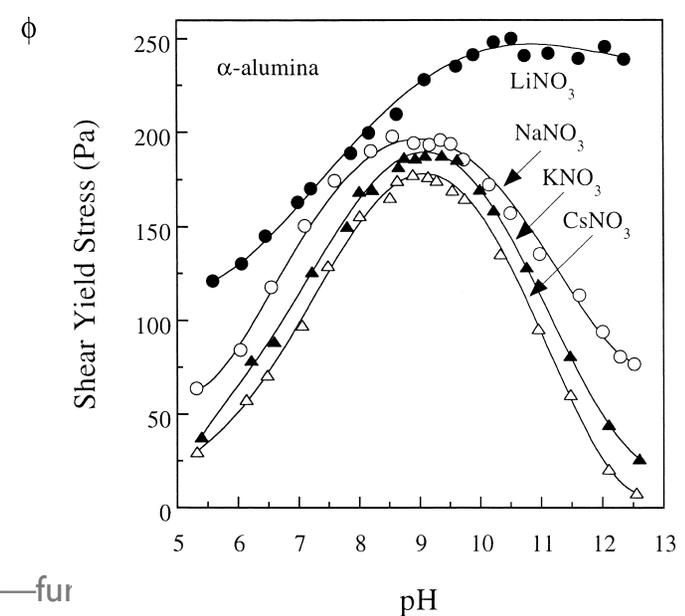
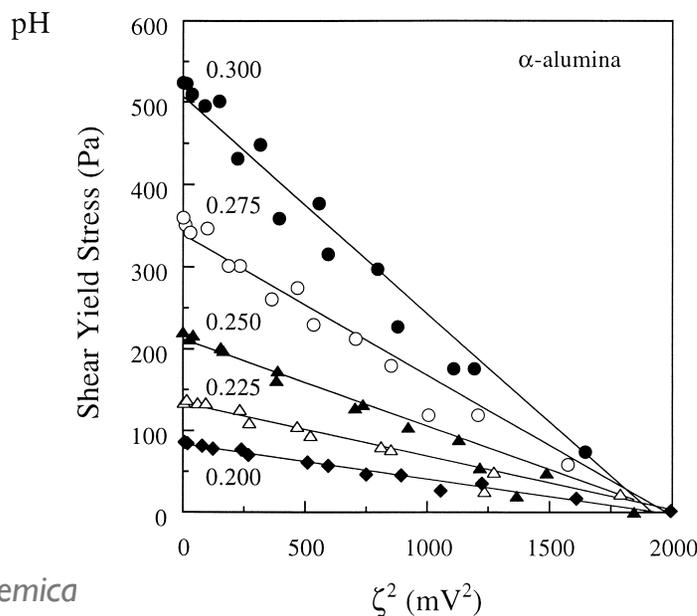
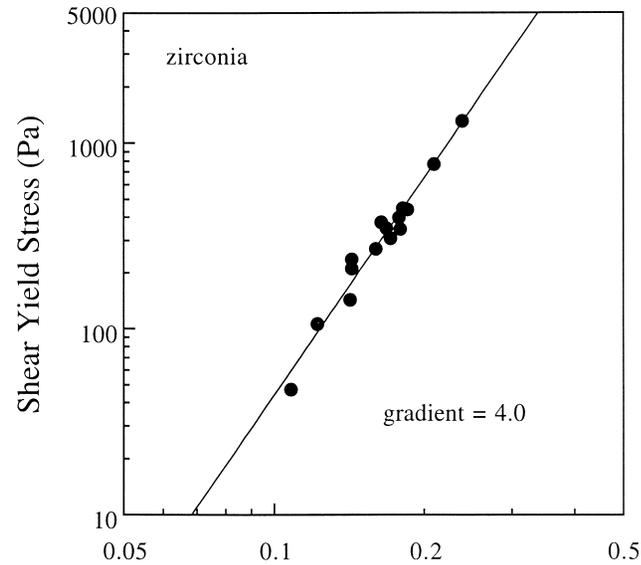
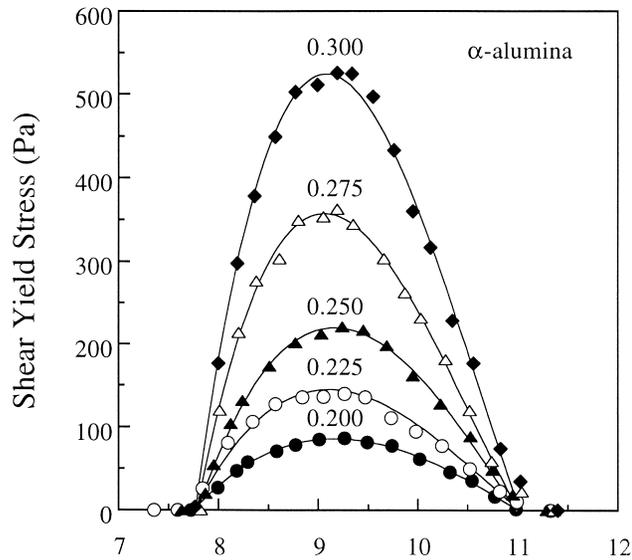
Rueb, C. J. & Zukoski, C. F., Viscoelastic properties of colloidal gels. *J. Rheol.* 41, 197–218 (1997).



$$G' \sim (\phi/\phi_G - 1)^p$$
$$p = 2.0 \pm 0.3$$

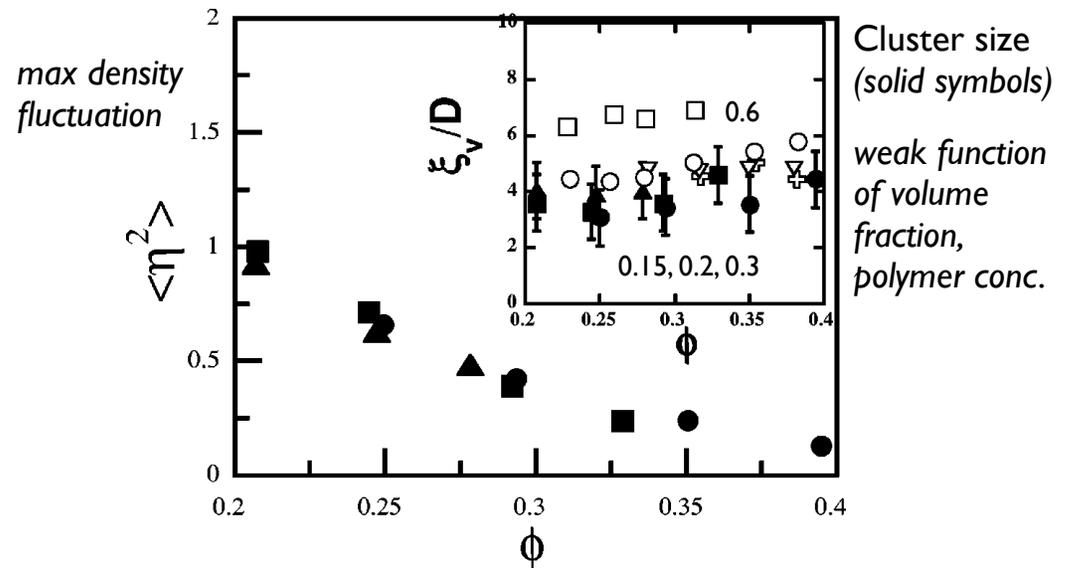
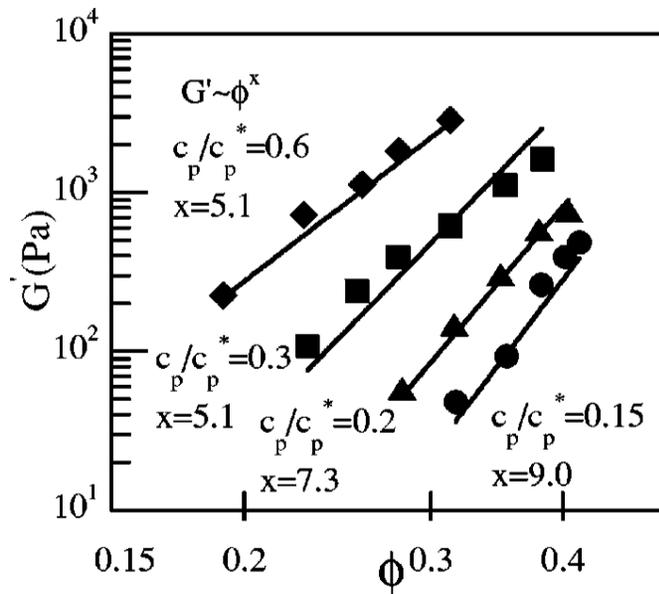
Mineral suspensions

Johnson, S. B., Franks, G.V, Scales, P.J. & Healy, T.W. "Surface chemistry–rheology relationships in concentrated mineral suspensions." *Int. J. Miner. Process.* 58, 267–304 (1999); Buscall, R., Ettelaie, R. & Healy, T.W., "Yield stress and contact forces in coagulated oxide dispersions." *J. Chem. Soc. Faraday Trans* 93, 4009–4015 (1997).



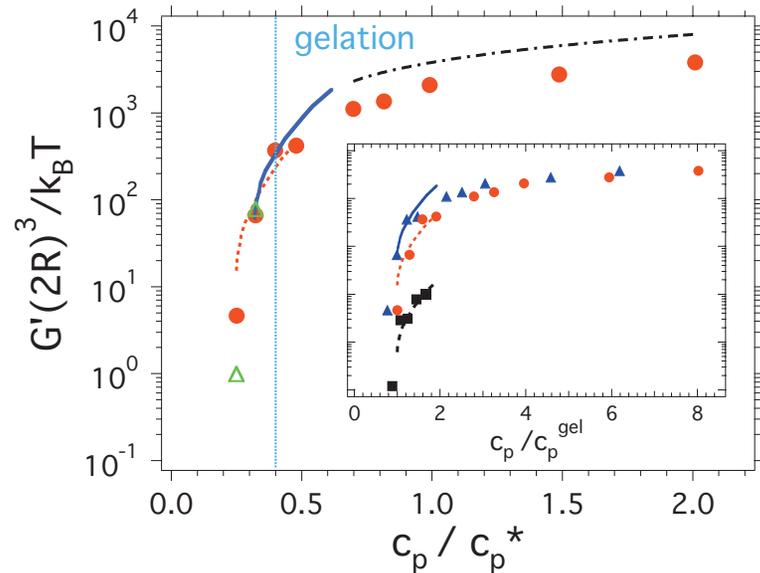
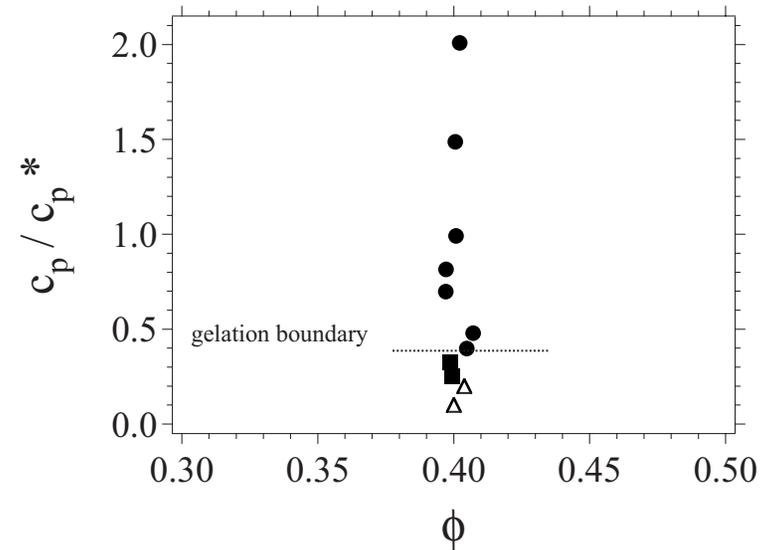
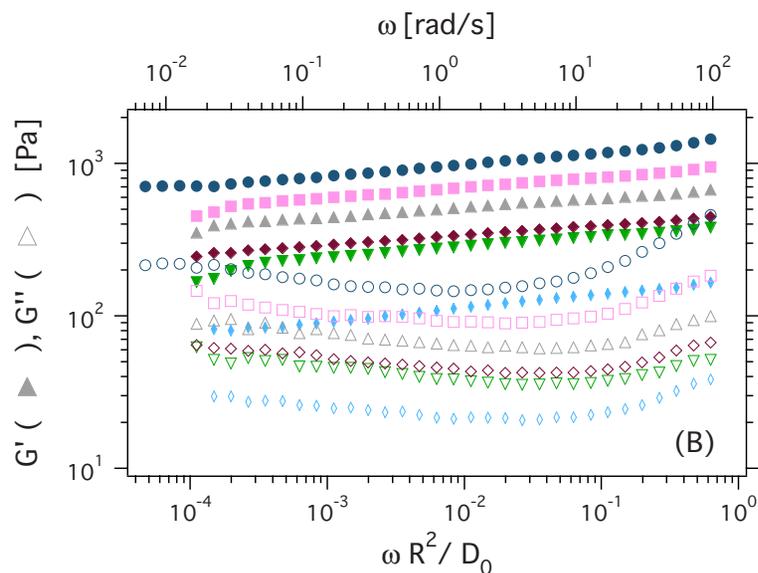
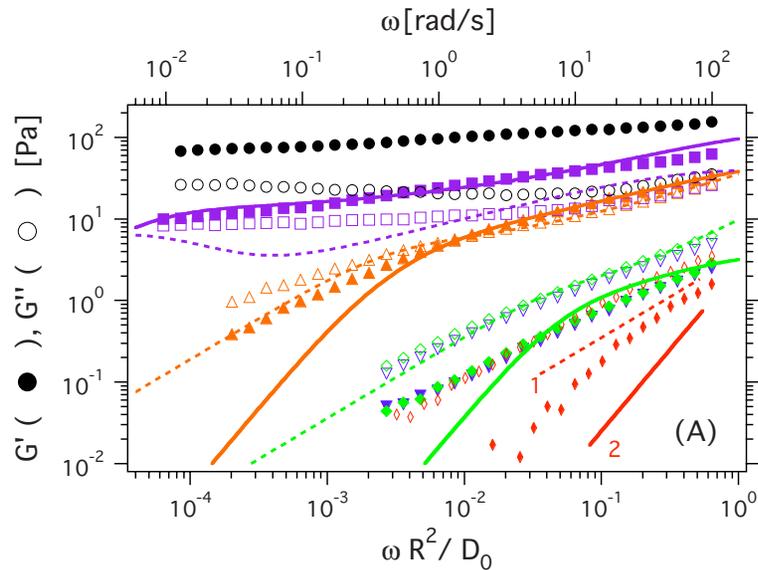
Depletion gel — silica, non-adsorbing polymer

H. K. Chan, et al., Phys. Rev. E, 85, 041403 (2012).
 N. Koumakis, et al., Soft Matter, 7, 2456-2470 (2011).
 S. Ramakrishnan, et al., Phys. Rev. E, 70, 040401 (2004).



Colloid-polymer mixtures

Laurati, M. et al. Structure, dynamics, and rheology of colloid-polymer mixtures: From liquids to gels. *J. Chem. Phys.* 130, 134907 (2009).



Gel rheology — challenges

- Poor reproducibility
- Sensitivity to method of preparation
- Sensitivity to shear history (thixotropic)
- Limited range of linear viscoelastic response
- Slip in rheometers
- Sedimentation